Give us a sign

by Professor E. R. Laithwaite*

An earlier article in this series[1] produced a lot of written discussion, some of which was published in these pages[2]. It was not, of course, unexpected. Indeed, in the article itselt I suggested that there would be demands comparable with those of some 2,000 years ago: "Give us a sign".

Now I thought that I had done this in revealing the amazing patent specification of one Henry Wallace of Missouri, but it obviously did not satisfy Mr Coates. And yet Ralph Coates and I have so much in common. There are sentences in his text that I shall use again many times in defence of my theories. Certainly they are worth repeating here as an introduction to some work which, had there not been a "letter from Isaac", might have remained dormant for some months.

In particular, consider his words concerning momentum and energy: "... both are concepts with specific definitions invented as a basis for book-keeping and as a means to help integrate one's view of the mechanical world ..."; later: "... provided you use my system of book-keeping"; and finally: "You must arrange your ground rules so that the answer is what you know to be right ..." is a statement that might more easily have come from Faraday than from Newton — it is the very bedrock of an engineer's belief.

If then I am allowed to have a system of book-keeping different from that of Sir Isaac, it should perhaps be more profitable to weigh it against the evidence (both experimental *and* theoretical) than to criticise it outright, as others (certainly not Mr Coates) have done on the grounds that Newton's Laws of Motion will *never* need amendment.

The new system of book-keeping

The equations of a gyroscope† are identical to the equations of an induction motor in two-axis form. By analogy we can relate quantities in the one discipline to their counterparts in the other. When an induction motor runs at the same speed as that of its field, v_s, it is said to be "at zero slip". At other speeds, v, we define "fractional slip" as $(v_s - v)/v_s$.

The Goodness of an induction motor is well known and defined in my original paper[3]. It is basically the ratio of the circulating energy (often coined as the "imaginary" part of the impedance) to the energy appearing as heat. In terms of impedance ratios the Goodness Factor $G = X_m/R$.

Now a gyro that is released from rest with one end of its shaft on a tower and the shaft initially horizontal, as in Fig. 1, can never acquire any angular momentum about a vertical axis, simply because there is no means of giving it any as it is released[4]. But throughout its subsequent nutation there are times when it has an obvious moment of inertia I about a vertical axis through the tower, as well as an even more obvious angular velocity Ω about that axis, but *zero* angular momentum. An electrical engineer has no difficulty in accepting such a situation and in expressing it in complex number form as "imaginary" momentum $jI\Omega$ as with $jL\omega$ or $1/jC\omega$ for electrical components L and C).

If this same technique is used for a gyro we can define the Goodness in terms of the $jI\Omega$ part of any precession divided by the *real* momentum $MR^2\Omega$ due to the rotation of a dead weight of inertial mass *M*, at radius R, rotating with the wheel. Thus in Fig. 1, a perfect gyro (i.e. one with all its mass concentrated in a rim of no thickness connected by spokes of zero mass to a shaft of zero mass) has a natural precession rate Ω_o given by:

$$M'gR = \omega I\Omega_o$$

where M' is the gravitational mass of the rim gyro, ω is angular speed on its own axis and I its moment of inertia (MR^2). But the gravitational mass M suspended from a hook at the end of the non-rotating (on its own axis) shaft will increase its precision rate by $(M + M')/M^1$ times.

At this new precession velocity Ω, the mass M, if suspended from a flexible joint or hook, will take up a position as shown and exhibit centrifugal force and real momentum about the axis of the tower. The imaginary momentum is located in the live wheel and the ratio of the two momenta is the Goodness Factor, G. The situation must therefore obtain if the increased precession rate is obtained in any other way, for example by applying a torque other than by gravity and resisting the attempt of the gyro to rise.

Gyro "slip"

Let us attempt to define the "slip" of a gyro in terms similar to those for an induction motor, thus: if a gyro precesses *naturally*, under a single torque applied about an orthogonal axis, its precession speed Ω_o corresponds to the field or synchronous speed of an induction motor. If, however, the gyro is forced to precess at a different speed Ω by a torque applied about the *same* axis, which may consist of a torque acting against inertia only (see next section), then the fractional slip, s, is defined as:

$$s = \frac{\Omega_s - \Omega}{\Omega_s}.$$

We see at once that s can be greater or less than unity and either positive or negative. This introduces all sorts of new

†My definition of a gyroscope is any spinning body whose axis of spin is capable of being rotated about a non-parallel axis. Precession is thus the opting for such motion and I would term this "rotation of a live body" as opposed to that of a "dead mass", c.f. an excited electric motor as opposed to an "unexcited" one.

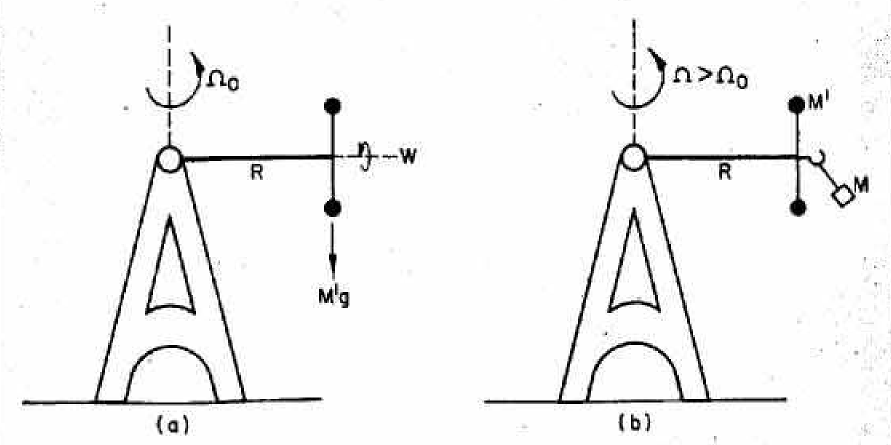

Fig. 1—(a) Precession of a perfect (rim) gyro. (b) The effect of adding a "dead" mass, M.

*Professor of Heavy Electrical Engineering, Imperial College, London.

concepts, such as negative centrifugal force. But first, let us see what known phenomena this new idea of gyro slip satisfies. To do this must inevitably involve G, as would any similar attempt to investigate a rather inferior induction motor (one with a low value of G). By using induction motor theory the following equation emerges for the inertial mass of the gyro whilst subject to forced precession:

$$M = \frac{s\,M_0(1 + G^2)}{1 + s^2G^2} \quad (1)$$

where M_0 is the inertial mass of a body which is in any condition other than spinning. (Although M_0 is always numerically equal to the gravitational rest mass M_0 defined by Einstein, it is incorrect to denote it by the same symbol because the two are not dimensionally equal and the numerical equivalence is always assumed.)

Equation (1) rests on the idea that if the Law of conservation of angular momentum is to be preserved, the value of the inertial mass must be allowed to change. The alternative is that if M is always to be identified with M then the law of angular momentum has to go!

Equation (1) tells us that when $s = 0$, $M = 0$. This implies a zero value of M at the natural precession rate of a perfect gyro (an idea not too difficult to accept). If the gyro is at rest precessionally, $\Omega = 0$ and $s = 1$, whence $M = M_0$ — which we know to be true. (Compare an induction motor at rest ($\omega = 0$), and the slip, $s = 1$. The secondary resistance per phase in the equivalent circuit is then simply its normal value R_2. At other speeds it is R_2/s.)

Rate of onset of torque

If a precessing gyro is "urged on" to precess faster and faster by an ever-increasing torque (rotational "surge") about the precession axis, s becomes negative and centrifugal force should also become negative. The introduction of a steadily increasing torque needs some further explanation with the help of Figs. 2 and 3. Fig. 2 defines the axis X, Y and Z which are used to label by suffices the various quantities such as angular deflection θ, angular velocity ω (or Ω) and angular acceleration Ω. Fig. 3 shows the quantities relevant to the argument all plotted against time.

Ignoring the transient that occurs at the first onset of the torque ramp in (a), and all its consequences, as one usually does in comparable electromagnetic phenomena, a torque ramp in the X-axis produces an appropriate precession ramp Ω_Y in the axis of y, as in (b). But if Ω_Y increases uniformly with time, there is a constant acceleration in y, $\frac{d\Omega_Y}{dt}$, which is of rectangular shape, during the time interval during which the $\frac{dT_X}{dt}$ was sustained, as in (c).

By ordinary (non-gyroscopic) dynamics this $\frac{d\Omega_Y}{dt}$ will produce a back reaction T_Y $(= I_Y \frac{d\Omega_Y}{dt})$ on the y-axis. Now a T_Y produces an Ω_X according to the equation: $\Omega_X = T_Y/I_Z\omega_Z$ (z being the precessing spin axis), and Ω_X accumulates deflection θ_X in the x-axis as Ω_X dt is integrated over the region $0 < t < t_0$. Thus a deflection is obtained *in the same axis* as the applied torque, which is not a usual region of gyroscopic action. θ_X is clearly calculable as:

$$\theta_X = {}_0\!\int^{t_0} \Omega_X dt = \Omega_X t_0 = \frac{T_Y t_0}{I_Z\omega_Z} = \frac{I_Y \dot\Omega_Y t_0}{I_Z\omega_Z} = \frac{I_Y k t_0}{(I_Z\omega_Z)^2}$$

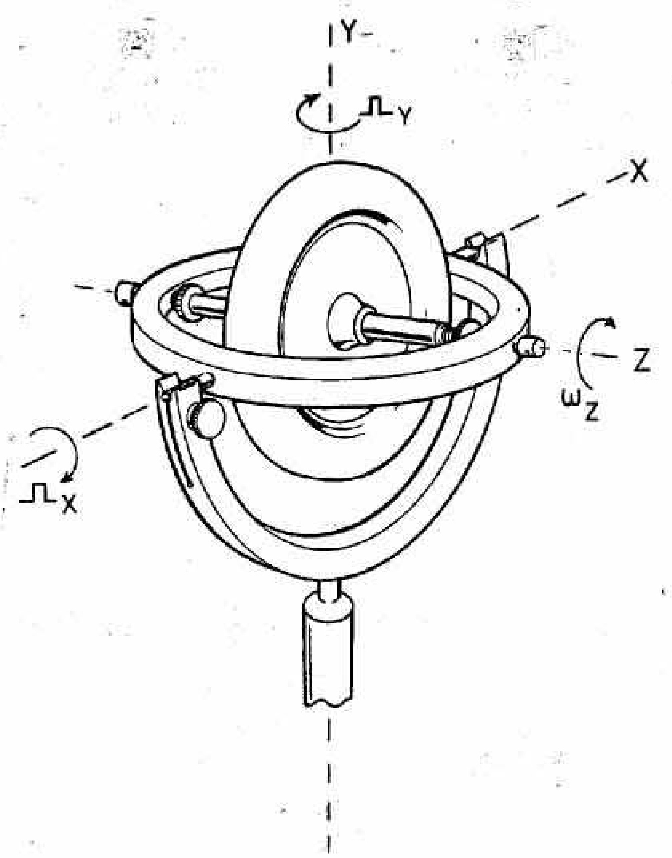

Fig. 2—Definition of axes for a gimballed gyro.

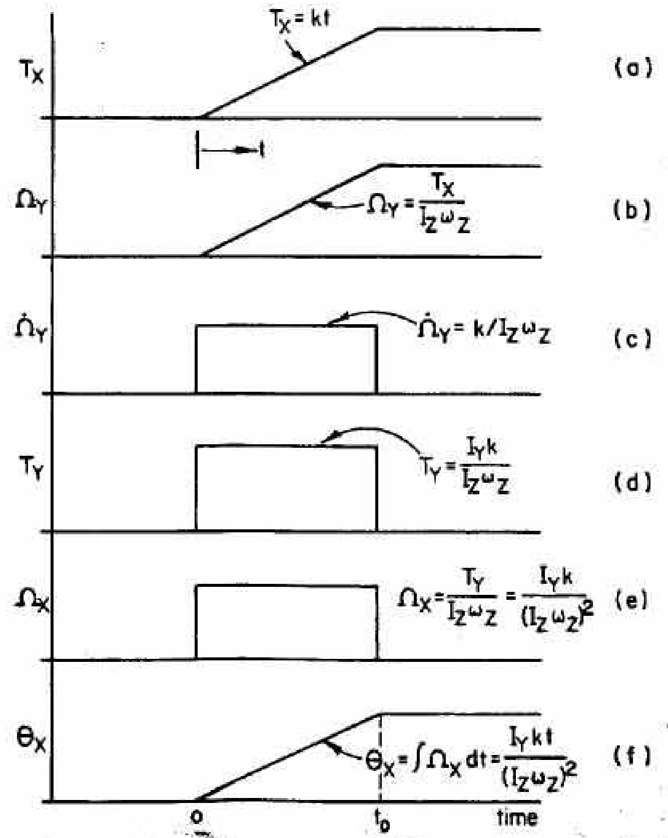

Fig. 3—Interactions between axes on application of a steadily increasing torque.

Of course the devoted will claim that all this was calculated using Newtonian formulae, and they would be correct. But the result is, to say the least, interesting, from at least two points of view.

Karate and similar phenomena

One of my earliest memories (I could have been no more than seven years old) was of being taken to see a travelling road show that came to the village where I lived and gave one performance nightly for a whole week. There was a conjurer, some dancers, a juggler, a piano accordionist (for community singing) and a strong-man act. The strong man was the strongest I have ever seen. He lifted two dumb-bells (each about 50kg) mounted on a leaded board (another 50kg at least which rested on his knees and chest when he was in the position known as the "wrestler's bridge". A long wooden plank was then added and eight local men sat four-a-side and the strong man did press-ups to the ground and back! For encores he lifted the grand piano, broke bricks with his hands alone and bent a length of horseshoe iron into a double loop.

His last act was to knock a six-inch nail through a four-inch block of wood in three hits, using only the palm of his hand! He crouched over the nail with his palm no more than half an inch above the nail head (it had been just tapped in to give it a start). He concentrated for a full half minute. Then his hand flew high in the air and we saw that the nail had penetrated some inch and a half into the wood. He repeated the process and on the third "hit" the nail came splintering through. I shall never forget my reaction to this feat. "He cheated", I thought, "he *pushed* it: he didn't *hit* it", never stopping to marvel at the feat however he did it.

But 50 years later I was to repeat his actions in trying to effect a θ_X from a T_X using a double-gimballed gyro, by the method indicated in Fig. 4. It is necessary to apply torque *only* about the horizontal axis (such as by pushing vertically downwards on the inner ring opposite a rotor pivot). Any torque about the vertical would produce inner ring precession and invalidate the experiment. This was made impossible by using a ball-point pen with which to press down on the smooth, horizontal upper surface of the rectangular-section inner ring, as shown. Even so, there are difficulties because, as Fig. 3(c) shows, a value of Ω_Y will accumulate Ω_Y (α t) and θ_Y (α t^2) and the ball pen must chase the contact point around or risk application of the increasing force at a reducing radius which also could invalidate the experiment. What is even more difficult is to continue to apply the peak force at the end of the ramp (after time t_o) and to avoid doing this I proposed to try to lift the pen off the ring in zero time — a negative impulse! Difficult as this is, it should be possible to do it in a much shorter time than t_o and so produce a different kind of return to the $t = 0$ condition which the gyro would accept as comparable to a switching-off transient in an electrical device.

This experiment calls for the utmost concentration. The instant the ball pen touches the ring, you must increase the force rapidly and as your strength reaches its limit, you must get off the ring so quickly that your hand flies up in the air in an attempt to produce the negative impulse. But if you practise, you can indeed produce a θ_X by a varying T_X. The technique is the same in Karate. To break a man's collar-bone you do not start the hand's movement from more than a foot above the target, yet mentally you must aim to reach maximum velocity at his navel. In this way, your hand will hit his collar with more than an acceleration f. It will have some $\frac{df}{dt}$ and possibly $\frac{d^2f}{dt^2}$ and it is this that will do the damage.

Fig. 4—Applying an increasing torque to the inner ring.

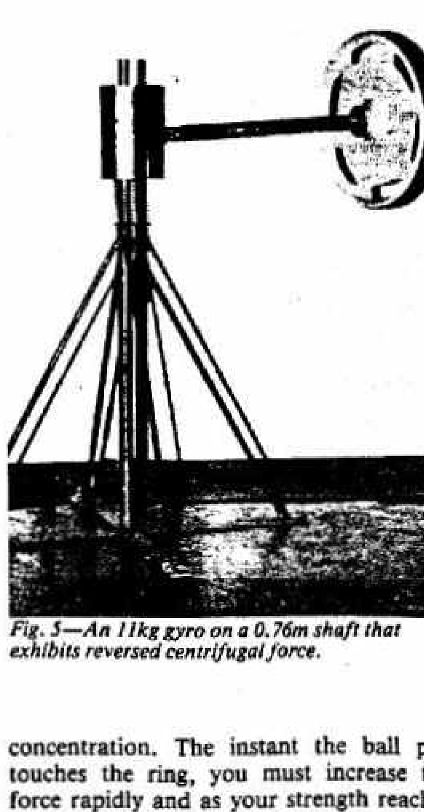

Fig. 5—An 11kg gyro on a 0.76m shaft that exhibits reversed centrifugal force.

There is a "pub legend" that the average man cannot break an egg placed on his hand with its long axis parallel to the fingers by closing the hand and squeezing. Mostly it works, to the amazement of myself when I saw a reasonably strong man try, and fail. But I realised that had he but dropped a 2p piece from a foot above the egg, edge on, the coin would have penetrated the shell easily. So what had the 2p piece got that human muscle had not? — the impulse, of course, the $k_1 \frac{df}{dt} + k_2\frac{d^2f}{dt^2}$. So I tried a *snatch*, Karate fashion, and succeeded at the first attempt. (Just how many other ceilings will now be spattered with raw egg as the result of this article, I shudder to think.)

Negative centrifugal force

The evidence for negative centrifugal force in the presence of accelerated precession I discovered in a disastrous experiment on Boxing Day, 1974 during a break in rehearsal for the fourth Christmas Lecture of that year at the Royal Institution. I was persuaded to take 0.3m wheel on a long shaft, on an iron tower (Fig. 5), back to Imperial College to get "a better picture" outdoors for a Press photographer.

On Boxing Day the College is like a morgue. The photographer insisted on photographing me from the top of a long flight of steps, so *he* had to spin up the wheel with me, using a hand drill, and then tear up the steps to operate the shutter. Meanwhile of course, the precessing gyro had begun to "droop" because of pivot friction at the top of the tower. But this was no problem: as the gyro passed me I leaned over the wheel and gave the shaft a hefty push to raise the shaft again. I *did* raise it, but the tower fell over and the wheel hit the lawn, gathered speed rapidly and passed by me like a greyhound, dragging its tower behind it. The base of the tower took with it a piece out of my trouser leg and a piece of the leg inside!

Whilst I was thinking ruefully over the event I asked myself "Why didn't I catch the shaft?" I knew it had no momentum to absorb. The answer came quickly: "I couldn't. The tower fell over *away* from me. The wheel shaft *pushed* it over. There was negative centrifugal force!" How could it take me so long to associate the strong man 50 years ago, the two-axis analysis of the induction motor and this painful experiment with the big wheel? Problems have a habit of looking like the north face of the Eiger until you solve them. Then they crumble into a heap of dust so small you wonder why you stumbled over it.

References

1. "Roll Isaac, roll Part 2", *Electrical Review*, 16 March, 1979.
2. Coates, R.: "I will not roll — yet, *Isaac Newton*", *Electrical Review*, 19 October, 1979.
3. "The goodness of a machine", *Proc. IEE*, March 1965, Vol. 112, pp. 538-541.
4. Maunder, L.: Letter to the Editor, *Engineering*, May 1975, Vol. 215, pp.388-389.